

Muon $g - 2$ and New Physics

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Abstract

Here we invoke the current and future perspective on muon $g - 2$ measurement when asking what the muon $g - 2$ could tell about the underlying structure concerning with the hierarchy problem. Here we take up two such models, the presence of which turns out to alter the standard model prediction for muon $g - 2$ significantly: one is TeV scale gravity scenario, the other supersymmetric model, in the latter case of which the precision measurement up to Z boson mass is taken into account as an explicit constraint.

1 Introduction

Further precise measurement of the anomalous magnetic dipole moment of the muon, conventionally denoted as $a_\mu \equiv \frac{1}{2}(g - 2)_\mu$, is now underway at Brookhaven National Laboratory (BNL). The perspective for the goal of this experiment is [1]

$$\Delta a_\mu(\text{expt}) = 4.0 \times 10^{-10}. \quad (1)$$

The recent report for its test-running course at BNL [1,2] combined with the previous one at CERN [3] gives to muon $g - 2$

$$a_\mu(\text{expt}) = 11659\,235\,(73) \times 10^{-10}, \quad (2)$$

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where the numerals in the parenthesis represent the uncertainty in the final few digits. Thus the precision (1) amounts to the determination of its value by one further digit.

Our primary interest is what we can learn when invoking such a improvement in muon $g - 2$ experiment. At present the standard model predicts

$$a_\mu(\text{SM}) = 11659\,160.5\,(6.5) \times 10^{-10}, \quad (3)$$

which includes the up-dated estimate on the leading hadronic vacuum polarization contribution [4], (See [5] due to analysis without recourse to τ decay data), and $\mathcal{O}(\alpha^3)$ contribution [6]. The designed precision (1) is put forward to find out the existence of the W , Z boson effect to muon $g - 2$ [7]

$$a_\mu(\text{weak}) = 151\,(4) \times 10^{-11}. \quad (4)$$

The electron $g - 2$ does not receive observable effects from Z and W bosons and are entirely saturated by QED effect. This fact enables us to find out the validity of calculation scheme of quantum field theory, including the perturbative renormalization procedure. Rather electron $g - 2$ provides the most accurate determination of the fine structure constant α [8] at present. From (4) and (1), the muon $g - 2$ is expected not only to obtain a conceivable evidence for such a structure about the electroweak interactions as involved in standard model, but also has the potential to indicate the existence of much richer ingredient associated with some theoretical problem.

When we incline to use muon $g - 2$ as a probe of new physics, the theme of the talk assigned to me as well, it would be important to recall the motivation or the merits of each model. Thus, here, I will focus on two concrete models considered to approach to "hierarchy problem", although it is somewhat a conceptual viewpoint. Here "hierarchy problem" stands for the following question in some narrow sense; what is assuring the stability of the electroweak scale, represented by W boson mass, M_W , against quantum fluctuation associated with high momentum modes below some cutoff scale. The cutoff scale here is the scale at which the gauge interactions appearing in standard model would become subject to some kind of modification. It may be the GUT scale M_G , at which the standard model gauge symmetry group is merged into a larger symmetry group. Or it may be the scale at which the gauge boson is resolved into more fundamental structure, for instance, into string.

Here we take up two models considered with such a motivation. One is TeV scale gravity discussed in the next section, and the other supersymmetry in Sec. 3. Sec. 4 concludes with remark on several facets for muon $g - 2$ to probe new physics practically.

2 TeV scale gravity

The laboratory experiments check the structure of gravity has been met up to the order of millimeters. With this in mind let us turn our attention to the scheme introduced a couple of years ago [9].

Let us imagine that our world is confined in a three-brane, the extended object with three-spatial directions, flowing in a higher dimensional space $((n + 4)$ -dimension in total). These n -directions are compactified to an n -dimensional torus with the same length $2\pi R$, for simplicity. Then there are an infinite tower of Kaluza-Klein states from the four-dimensional space-time point of view. They are the states with non-zero momentum in the extra directions. In four-dimensional world such a momentum appears as the mass, the scale of which is characterized by inverse of R .

The behavior of static potential between two point-like sources prepared with separation r illustrates an important aspect of the present setting [10]. When r is large compared to R , the potential behaves as $\sim 1/r$, nothing but the form of the usual Newtonian one. The Planck scale $M_{\text{PL}} \simeq 10^{19}$ characterizes its strength. However, once the separation r reaches below R , the probability that Kaluza-Klein states get excited cannot become neglected any more. Thus counting those states which propagate essentially like massless states between two sources leads $r^{-(n+1)}$ as r -dependence of the potential at short distance, characteristic of $(n + 3)$ -spatial dimension. This only reflects the fact that the local structure less than the compactification scale is that of $(n + 4)$ -dimensional space-time. A noteworthy point is that the strength of force is then characterized by another Planck scale, M_* ;

$$M_* = \left(\frac{M_{\text{PL}}^2}{R^n} \right)^{(n+2)}. \quad (5)$$

This is the strength of the gravitational interaction in the bulk theory, the fundamental Planck scale.

Now we take R equal to 1 mm², which corresponds to the length scale one-order less than the current reach of the experiment on gravity. With two extra compactified directions $M_* = 1$ TeV³.

If the three-brane were further of Dirichlet-brane type, on which the open string can attach, our standard model gauge bosons would become the tan-

² Energy scale is related to the length scale L through

$$E = 0.197 \text{ eV} \times \frac{1 \text{ mm}}{L}. \quad (6)$$

³ It was commented by A. Kataev in this workshop that consideration on the effect to the life-time of the red giant stars appears to reject the possibility, $M_* = 1$ TeV, which he heard at the seminar by Arkani-Hamed [10].

gential component of the ground state of such an open string. Then the gauge bosons would get resolve into strings higher than 1 TeV as long as the string coupling constant is of order unity. Thus there is no hierarchy problem ab initio.

At present we are lacking the precise formulation of theories and the detail knowledge on its dynamical aspects (especially on the compactification mechanism). However the dimension counting argument with symmetry consideration has been an enough tool when we estimate the order of magnitude about some effect in low energy phenomena unless the effective theory description breaks down, although we rather expect to superstring theory that many miracles beyond this assumption occur.

The argument begins with the chiral symmetry for muon. When muon was massless, the magnetic dipole coupling would be absent. Thus the magnitude of magnetic moment will be proportional to muon mass m_μ and the corresponding operator appears in the effective Lagrangian:

$$\mathcal{L} = e m_\mu A \bar{\mu} \sigma^{\lambda\rho} F_{\lambda\rho} \mu. \quad (7)$$

with the the extra effect A due to the new structure characterized by mass scale M_* . Since the mass dimension of A turns out to be minus one in four space-time dimension, the dimension counting now gives

$$A = c \times \frac{1}{M_*}, \quad (8)$$

with the numerical constant c of order unity. Insertion of (8) into the effective interaction (7) with a slight rearrangement yields

$$\mathcal{L} = \frac{e}{4m_\mu} \left[4c \left(\frac{m_\mu}{M_*} \right)^2 \right] \bar{\mu} i \sigma^{\lambda\rho} F_{\lambda\rho} \mu. \quad (9)$$

The quantity found in the square bracket of the above expression corresponds to the additional contribution to a_μ due to the presence of TeV scale gravity. Thus the additional effect to a_μ can be read off as

$$\delta a_\mu = 4c \left(\frac{m_\mu}{M_*} \right)^2 \times 10^{10}, \quad (10)$$

which becomes for $M_* = 1$ TeV

$$\delta a_\mu \sim (4c \times 100) \times 10^{-10}. \quad (11)$$

Thus a crude estimate shows that the effect from TeV scale gravity is in the marginal situation to be detected even with the current accuracy (2). The

future accuracy (1) is quite adequate to detect the existence of new aspect of gravity characterized by TeV scale ⁴.

3 Supersymmetric Model

Now we will take our attention to the search of supersymmetry with the use of muon $g - 2$, which has been discussed as a machinery to check the various advocated models or from some generic standpoint [12,13,14,15]. The detail formula and so on in this section will also be found in a separate literature [16].

In the narrow sense of the "hierarchy problem" defined in the introductory remark, supersymmetry makes it possible to extend the standard model gauge group into the larger gauge group in grand unified theory (GUT). This is realized in the form of the cancellation of the dangerous quantum corrections within each supermultiplet. The bosonic partner of muon, for instance, has not been observed yet so that it must be much heavier. Smuon can be let heavier by giving it a lifting-up mass, m_S . This procedure does not spoil quantum stability as far as m_S is not so apart from the electroweak scale. The scale m_S works as the ultraviolet cutoff scale for the phenomena accessed by low momentum probe while it works as the infrared cutoff from the supersymmetric high-energy side. It has been recognized that unification of the coupling constants of three gauge interactions, $SU(2)_L$, $U(1)_Y$ and color $SU(3)_C$, is achieved by assuming the particle content of the minimal supersymmetric extension of standard model above such m_S . For the future purposes it deserves to recall here that the knowledge from precise measurement around Z pole plays an indispensable role to establish this fact.

Now we examine the effect on muon $g - 2$ induced from supersymmetric theories. They come essentially from two diagrams. One is the chargino-sneutrino loop, where the charginos are the admixture of $SU(2)$ gaugino \tilde{w}^- and the charged Higgsino. The other is the neutralino-smuon loop, where the neutralinos are the admixture of their neutral counterparts. The other contributions, such as the charged Higgs loop one, are so small that they are irrelevant even for future study.

Fig. 1 is intended to demonstrate what magnitude of those effects to muon $g - 2$ is expected. The figure shows the supersymmetric contribution to muon $g - 2$ as a function of μ parameter (the supersymmetric mass common to two Higgs supermultiplets), for relatively large $\tan\beta$ (the ratio of the vacuum expectation values of two Higgs doublets), the supersymmetry breaking slepton mass set equal to 200 GeV, and for three choices of supersymmetry breaking

⁴ During the preparation of the talk, I have noticed that more concrete demonstration has been performed in a similar context [11]

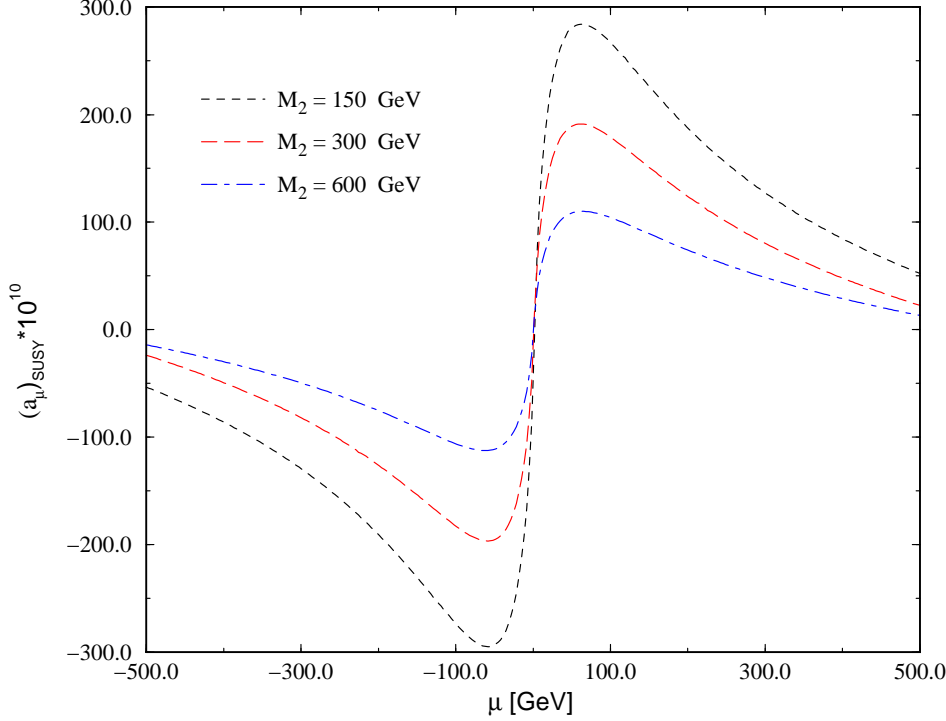


Fig. 1. μ dependence of $(a_\mu)_{\text{SUSY}}$ for $\tan\beta = 50$ and $m_{\tilde{\mu}_L} = m_{\tilde{\mu}_R} = 200$ GeV.

$SU(2)_L$ gaugino mass M_2 with $U(1)_Y$ gaugino mass given here through the GUT relation

$$M_1 = \frac{5}{3} \tan^2 \theta_W M_2. \quad (12)$$

As the weak effect is 15×10^{-10} , the supersymmetric effect can become substantial. Actually the muon $g-2$ even with the current accuracy excludes the region of negative sign of μ for this set of the other parameters.

Fig. 1 is drawn without paying any attention to the other constraints on supersymmetric models already present. The direct search of superpartners of the known species puts the lowest bound (93 GeV) to the lightest chargino mass, and the bound (78 GeV) to the mass of each lighter slepton [17]. In fact the chargino mass bound requires that the absolute magnitude of μ parameter be greater than about 100 GeV for the gaugino mass in our interest, while the slepton mass bound demands $|\mu|$ less than about 400 GeV. With those regions excluded Fig. 2 shows the contours each of which has equal magnitude of muon $g-2$ on the μ - M_2 plane. Since the future accuracy is much smaller than the interval between the neighboring contours in Fig. 2, we have a great chance to observe a signal coming from the existence of supersymmetry through muon $g-2$.

Now we are tempted to grasp the specific feature of muon $g-2$ in search of supersymmetric theory. It will turn out that muon $g-2$ seems to have a peculiar property which is not shared by any other observables.

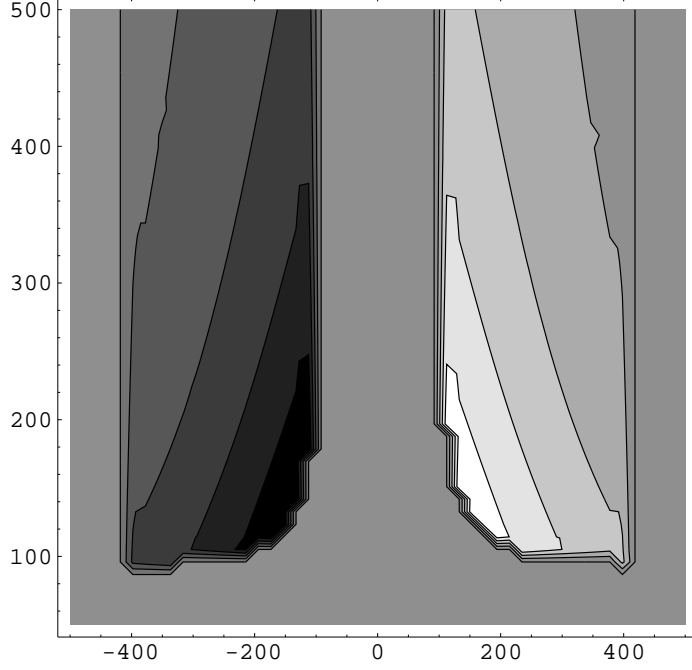


Fig. 2. Contours with equal $(a_\mu)_{\text{SUSY}}$ in μ (horizontal direction)- M_2 plane for $\tan \beta = 50$ and $m_{\tilde{\mu}_L} = m_{\tilde{\mu}_R} = 200$ GeV. The two islands are the regions escaping from any other constraints. The contours are drawn with the interval of 50×10^{-10} from -200×10^{-10} to 200×10^{-10} for $(a_\mu)_{\text{SUSY}}$. Darker face corresponds to smaller $(a_\mu)_{\text{SUSY}}$.

In the most honest region of parameter space $|\mu| \geq (2 \sim 3) M_2$, chargino-sneutrino loop contribution dominates over neutralino-smuon loop contribution. The current basis analysis helps one to catch up with the qualitative dependence on the various parameters. As long as $\tan \beta \geq 3$, the chirality flips due to the vacuum expectation value $\langle H_U \rangle$ of the Higgs field giving mass to the up-type quarks, turning \tilde{w}^- to the charged component of \tilde{H}_U , which transformed to the charged \tilde{H}_D due to μ -term. Picking $\sin \beta$ from $\langle H_U \rangle$ and $1/\cos \beta$ from a yukawa-type coupling involving muon, the dominant contribution in the present situation becomes

$$(a_\mu)_{\text{SUSY}} \propto +\mu \tan \beta, \quad (13)$$

although the overall sign needs a detail computation. From this expression we can read off such properties that

- (a) The effect to muon $g - 2$ is greatly enhanced for large $\tan \beta$ [12]. In fact, when $\tan \beta$ is small, the overall magnitude of SUSY effect is drastically reduced as shown in Fig. 3. Thus in this case the current experiment could not put any restriction on its existence. But the future accuracy in

- muon $g - 2$ is quite sufficient to explore it ⁵.
- (b) The sign of this contribution is govern by the sign of μ .

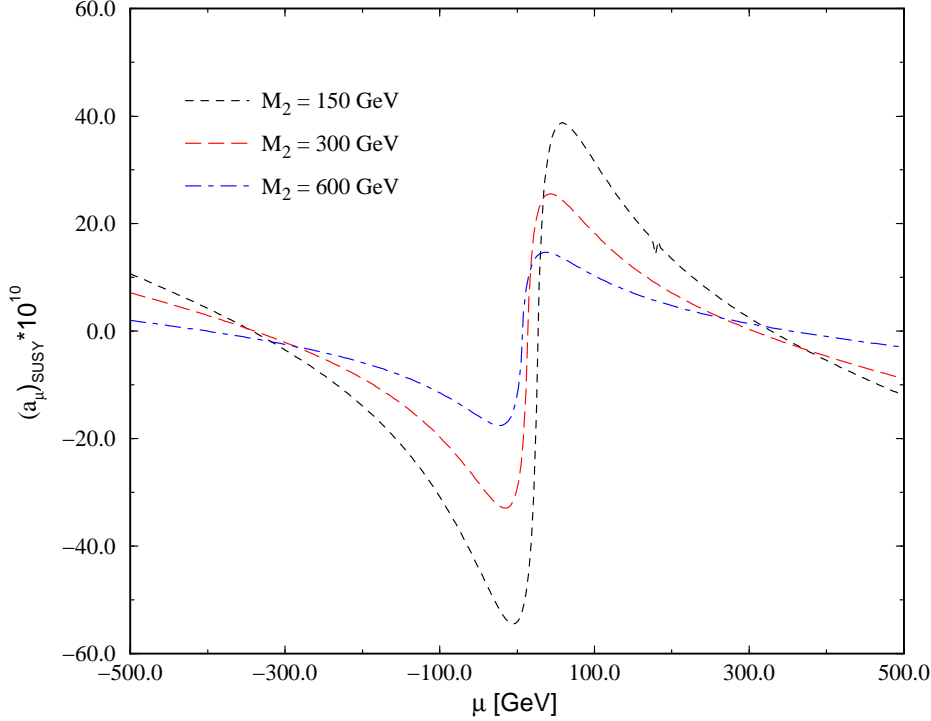


Fig. 3. μ dependence of $(a_\mu)_{\text{SUSY}}$ for $\tan \beta = 3$ and $m_{\tilde{\mu}_L} = m_{\tilde{\mu}_R} = 100$ GeV.

It is interesting to remind that a large $\tan \beta$ is a natural consequence of the gauge mediated SUSY breaking scenario, and elaborate analysis on muon $g - 2$ has been performed in this line [15]. Or it is a necessary ingredient for unification of all yukawa coupling constants of the third generation.

As observed at both ends of μ -direction in Fig. 3, supersymmetric effect to muon $g - 2$ does not decouple even if we can let the absolute magnitude of μ large while those other parameters remain fixed. Note that small $\tan \beta$ case allows relatively large absolute magnitude of μ parameter without conflicting with slepton mass bound, as the mixing between left- and right-sleptons are proportional to a combination $\mu \tan \beta$.

Such a phenomenon can be understood from the following observation. When $|\mu|$ is large the chargino-sneutrino effect decouples, but the neutralino-smuon effect increases. Let us consider a diagram in which the chiral flip occurs due to the mixing between the left and right-handed smuons in the current eigenbasis. As the Higgsino does not propagate, suppression factor due to the inverse power of μ is now absent. Thus $(a_\mu)_{\text{SUSY}}$ becomes proportional to $-\mu \tan \beta$. (The sign is also opposite to the chargino-sneutrino effect (13).) This is the

⁵ The renormalization group analysis shows that QED correction tends to decrease those new effect about 6%, and this fact should be recalled at the critical stage of confronting with the experimental data [18].

reason for such a behavior in this large $|\mu|$ region.

From those observations, muon $g - 2$ seems to play the major role to find the sign and the magnitude of μ term. As far as I know, such a property sensible to μ is not shared by any other observables. Recall the following two facts, that is,

- (a) μ is a supersymmetric parameter which is associated with common mass of Higgs supermultiplet. Thus this is a parameter independent of the supersymmetry breaking parameters by nature.
- (b) Although supersymmetry assures the quantum stability of electroweak scale, supersymmetry does not set μ to this region at tree level automatically. This is the most annoying matter called as “ μ problem”. This problem stands out especially in the context of GUT.

Thus, once supersymmetry is established also by the other experiments, the determination of μ parameter through muon $g - 2$ may develop further theoretical access to the origin of μ , the origin of electroweak scale.

Before addressing to the future testing possibility in the small $\tan\beta$ regime, we remind the additional constraints implied from precision measurement at Z pole. As was mentioned, the result of this precision measurement has given an indispensable information to argue grand unification. It also has killed the naive technicolor models. Thus we should discuss the effect on muon $g - 2$ on the region of the parameter space consistent with those measurements.

They are summarized by four parameters. Three of these parametrizes the “oblique” corrections from new physics, with respect to “reference” standard model; here we take the one specified by

$$m_t = 175 \text{ GeV}, \quad M_H = 100 \text{ GeV}. \quad (14)$$

The last one is associated with the modification of coupling of bottom quark to Z boson. This is neglected here by assuming that the squarks are so heavy enough that their effects decouple. Since it has been recognized that the SUSY effect to W boson mass and coupling of τ to Z is not relevant within the current accuracy[19], we concentrate on S and T parameters.

Fig. 3 ⁶ shows a constraint implied from S and T parameters ⁷. The reference standard model is at the origin on this plane located in the contour of 90 % confidence level. The slepton contribution brings S parameter to negative, while the chargino and neutralino ones to positive. A set of two lines in the left-hand side pursues the response of the slepton effects to the change of SUSY breaking slepton mass for two values of $\tan\beta$ (solid line for $\tan\beta = 3$, dashed line for $\tan\beta = 50$.) The one on the right-hand side follows the response of the chargino effects against the change of μ parameter. The solid line with the

⁶ The author thanks G. C. Cho for drawing this figure several times.

⁷ Both axes are essentially S, T themselves here.

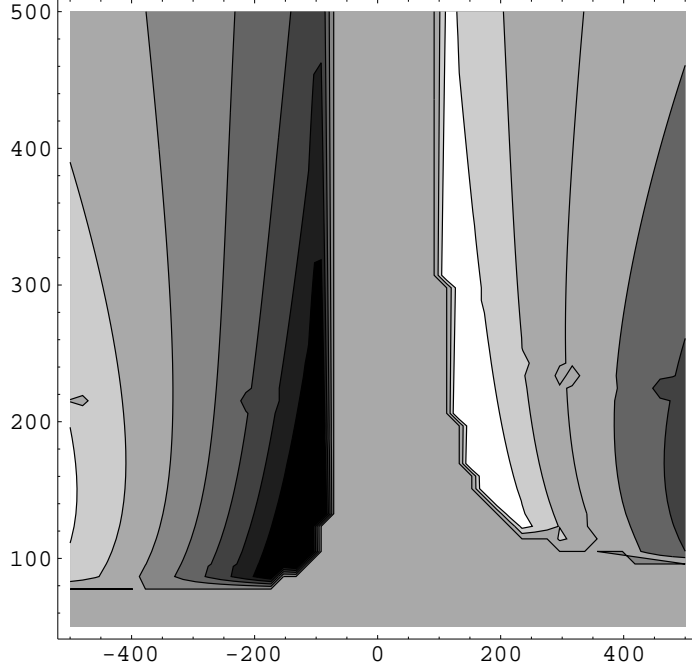


Fig. 5. Similar contours with equal $(a_\mu)_{\text{SUSY}}$ in μ - M_2 plane for $\tan\beta = 3$ and $m_{\bar{\mu}_L} = m_{\bar{\mu}_R} = 100$ GeV. The contours are drawn with the interval of 5×10^{-10} between -25×10^{-10} and 10×10^{-10} for $(a_\mu)_{\text{SUSY}}$.

dominated by the leading order QCD contribution which arises through the hadronic vacuum polarization. Its improvement is now awaiting for the precise knowledge about the low energy hadron production cross section planned to be accumulated at Novosibirsk, Frascati and Beijing.

The hadronic light-by-light scattering effect [20], which requires purely theoretical evaluation, may become an obstacle. Thus the reduction of its error also needs further challenge.

To summarize we discussed the effect to muon $g - 2$ from two candidates of models each of which accesses to “hierarchy problem”. We found that the potential signatures are expected from the existence of both two candidates by future measurement of muon $g - 2$ even on account of the precision measurement at Z pole. But this program cannot be accomplished without improvement in measurement of the hadron production cross section in low energy domain.

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